Additional example of Queueing

(I considered this one too long to leave in the text – RHV)

Example 2: Taxi Service

This is a queuing analysis to determine the number of taxis needed to serve the demand in a small town. Here are the assumptions for this example: Average time serving (including going to pick up point) - 20 minutes. Average time between calls - 6 minutes. Cost of driver & auto operation - $25/hour. Average fare & tip - $16. Customer willingness to wait - after 10 minutes, 50% will take a bus.

In setting up the simulation, the analyst could assume a call every six minutes. That table would look like this for 1, 2, 3 or 4 cabs in service:

Time	Start	Stop	Wait - 1	Wait - 2	Wait - 3	Wait - 4
00-06	0	20	No	No	No	No
07-12	7	26	--	No	No	No
13-18	13	32	--	--	No	No
19-24	19	38	--	--	--	No
25-30	25	44	No	--	--	--
31-36	31	50	--	No	--	--
37-42	37	56	--	--	No	--
43-48	43	62	--	--	--	No
49-54	49	68	No	--	--	--
55-60	55	74	--	No	--	--

And the pattern would continue to repeat. If calls come exactly every 6 minutes and each service takes exactly 20 minutes, 4 cabs will be just right. No one ever has to wait.

But how about if calls come in randomly? To simulate this, roll a di (one half a pair of dice), and each time a "1" comes up, we say a call has come in. And, instead of each trip being 20 minutes, we use a random method to determine how long the trip will be. Roll 6 dice, add them up and subtract one. That would give an average of 20, but with results ranging from 5 to 35. Or maybe the manager or analyst could keep track for several nights or check historical dispatch records. However it is done, we want to test the assumption that four cabs is adequate. We know three is not, of course. A random number table could also be used to construct this (see the Appendix to the book).

Now some reality has been introduced to the model. The following table was constructed by actually tossing one and six dice. Instead of the smooth flow we had by using only averages, you now see random idle servers and random waits.

In order to keep the table from going on for several pages, all times during the two hour simulation in which no change happens were eliminated. Here's what the headings mean:

- Time is in minutes from zero. The first twenty minutes are needed to get the system up and running, so they are not shown. The two hour simulation runs from minute 21 through 140.
- The Yes/Time column indicates that a call was received for service in that minute, and the length of the required service is indicated in minutes.
- Cab 1 busy indicates (with an x) that the first cab is in service; "Cab 2" the second, and so on. The numbers under the cab columns indicate that a trip started and how long it will take to complete.
- Unshaded boxes indicate that the cab is not busy; lightly shaded that it is busy; darker shaded that it is busy and has another fare waiting.

At least one taxi is idle from the start of the study at minute 21 through minute 45. After that point, there is NO idle time clear through minute 93, then it eases up again. At one point (minute 71) three customers are waiting at the same time.

Five customers over the two hours have waits of 9, 14, 18, 7 and 2 minutes. How much of a problem is this? The results of this model: In the 120 minutes 17 calls were received. 12 were served immediately and three more within ten minutes. One additional cab would have reduced all waits to within the ten minute limit (calculation not shown). That would have "saved" one of the two customers who would have taken a bus. The gain in this simulation of adding another cab would be $16 (one more

customer) but the cost would be $50 (2 hours @ $25/hr), for a net loss of $34. If the decision is strictly financial, running four cabs is best.

If the company wants to have a ten minute guarantee, a fifth cab is needed, but that will reduce profits overall. A compromise could be what some of the restaurant chains have done with their ten minute lunch guarantees: If we're not there in ten minutes, your trip is free. That way, it would only cost them $32 instead of $34. Yes, the restaurants that make that offer probably did this sort of an analysis to determine how many cooks and servers to have on duty.

Minutes	Yes / Time	Cab 1 busy	Cab 2 busy	Cab 3 busy	Cab 4 busy
21	no	x	end	NO	NO
24	x-32	x	NO	NO	start 24-55
27	no	end	NO	NO	x
41	x-33	start 41-73	NO	NO	x
42	x-21	x	start 42-62	NO	x
46	x-31	x	x	start 46-76	x
47	x-17	x	x	x	x + wait
49	x-9	x	x + wait	x	x + wait
55	no	x	x + wait	x	end + wait
56	x-21	x + wait	end + wait	x	start 56-72
63	no	x + wait	start 63-71	x	x
65	x-13	x + wait	x + wait	x	x
71	x-19	x + wait	end + wait	x	x + wait
72	no	x + wait	start 72-92	x	end + wait
73	no	end + wait	x	x	start 73-85
74	no	start 74- 92	x	x	x
76	no	x	x	end	x
77	x-17	x	x	start 77-93	x
85	no	x	x	x	end
89	x-24	x	x	x	start 89-112
92	no	end	end	x	x
93	x-16	start 93-108	NO	end	x
96	x-10	x	start 96-105	NO	x
105	no	x	end	NO	x
108	no	end	NO	NO	x
112	x-25	NO	NO	start 112-136	end
123	x-28	NO	start 123-150	x	NO
133	x-15	start 133-147	x	x	NO
136	no	x	x	end	NO
139	x-23	x	x	NO	start 139-161